The UNNS Tensor Protocol (UTP): From Recursive Vectors to Recursive Tensors

UNNS Research Notes

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Abstract

The Unbounded Nested Number Sequence (UNNS) substrate has been equipped with a vector representation through the UNNS Vector Protocol (UVP). We now extend this framework to the *UNNS Tensor Protocol* (UTP), which formalizes higher-rank recursion by mapping nests into tensor spaces. This generalization allows the definition of multilinear operators, tensor contractions, and curvature-like structures, bringing UNNS closer to physical field theories such as electromagnetism and Yang–Mills theory.

Contents

1 Motivation

Recursive sequences often interact in pairs, triples, or networks. The UVP provides a linear structure but is limited to rank-one vectors. The UTP generalizes UVP into higher tensor ranks, enabling:

- Encoding of *interacting nests* as tensor products,
- Definition of *recursion curvature* via commutator tensors,
- Bridging to physics models where fields are inherently tensorial.

2 Tensorization of Nests

Definition 2.1 (Nest Tensorization). *Let $\mathcal{N}_1, \ldots, \mathcal{N}_r$ be admissible nests. Their tensorization is defined by*

$$T(\mathcal{N}_1, \ldots, \mathcal{N}_r) = V(\mathcal{N}_1) \otimes \cdots \otimes V(\mathcal{N}_r) \in \mathbb{V}^{\otimes r},$$

where V is the UVP vectorization map.

Remark 2.2. *The rank r of the tensor corresponds to the number of interacting recursive layers.*

3 Protocol Rules for Tensors

The UTP extends UVP rules:

1. **Embedding:** Any collection of nests embeds into $\mathbb{V}^{\otimes r}$.
2. **Operator Action:** UNNS operators act multilinearly on tensors.
3. **Contraction:** Recursion contraction reduces rank, analogous to trace.
4. **Curvature:** Non-commutativity of operators induces curvature tensors.

4 Operator Actions on Tensors

Proposition 4.1. *Let $\mathcal{O}$ be a UNNS operator. Then*

$$\mathcal{O} \cdot (v_1 \otimes v_2 \otimes \cdots \otimes v_r) = \sum_{j=1}^{r} v_1 \otimes \cdots \otimes \mathcal{O}(v_j) \otimes \cdots \otimes v_r.$$

Remark 4.2. *This defines operator actions as derivations extended to tensors.*

4.1 Contraction

Definition 4.3. *Given a bilinear form $\langle \cdot, \cdot \rangle$ on $\mathbb{V}$, the contraction of a rank-2 UNNS tensor is*

$$\mathrm{Tr}(v \otimes w) = \langle v, w \rangle.$$

4.2 Curvature Tensor

Definition 4.4 (Recursion Curvature). *Given two operators $\mathcal{O}_1, \mathcal{O}_2$, the curvature tensor is*

$$\mathcal{F}(v) = \big(\mathcal{O}_1\mathcal{O}_2 - \mathcal{O}_2\mathcal{O}_1\big)(v),$$

which extends naturally to tensors.

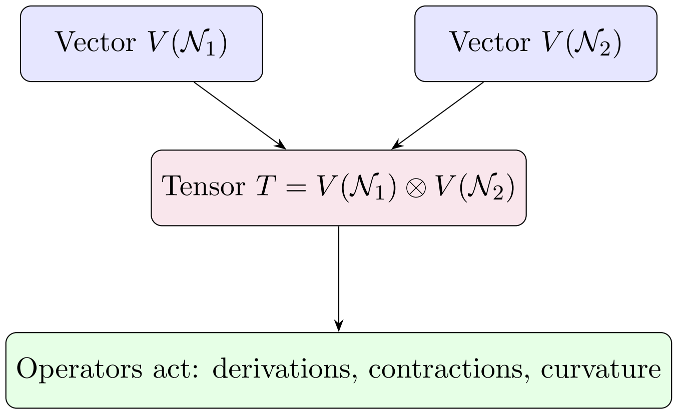
5 Theorems on Tensor Structure

Theorem 5.1 (Tensor Closure)**.** *The space* $\mathbb{V}^{\otimes r}$ *is closed under operator actions of the Dodecad.*

Proof. Each operator acts linearly on $\mathbb{V}$. By multilinearity, their action extends to $\mathbb{V}^{\otimes r}$. □

Lemma 5.2 (Curvature nontriviality)**.** *If two operators* $\mathcal{O}_1, \mathcal{O}_2$ *do not commute, the recursion curvature tensor* $\mathcal{F}$ *is nonzero.*

6 Diagrammatic Overview



7 Applications

7.1 Mathematics

- Generalizes linear recursion to multilinear recursion.
- Defines recursion curvature analogous to differential geometry.

7.2 Physics

- UNNS curvature parallels field strength tensors in gauge theory.
- Contraction rules resemble stress-energy tensors in relativity.

7.3 Computation

- Enables higher-order recursion simulators.
- Provides tensor-network interpretations of recursive processes.

8 Conclusion

The UNNS Tensor Protocol (UTP) extends the vector framework of UVP to higher-rank recursive interactions. This creates a bridge to tensor calculus, curvature, and gauge theory, establishing UNNS as a candidate substrate for recursive physics and computation.